9.4 Systematic Fault Analysis Using Bus Impedance Matrix

In the previous analysis we employed the Thevenin model and found the Thevenin voltage and impedance by means of circuit manipulation. That method works for small power circuits only: It is impractical for a real large power system. A systematic method is presented in this section which applies to power systems of any size. The method can also be programmed on a computer.

Consider a typical bus of an \( n \)-bus power system network as shown below. Here we show buses \( i \) and \( k \). The generator shown is a voltage source behind a reactance which may be \( X_*, X'_*, \) or \( X_d \). Transmission lines are represented by their equivalent \( \pi \) model and all impedances are \( \text{pu} \) on a common MVA base. A balanced three-phase fault is to be applied at bus \( k \) through an impedance \( Z_f \). The prefault voltages are obtained from a power flow solution and represented by the vector

\[
V_{bus} (0) = \begin{bmatrix}
V_1 (0) \\
\vdots \\
V_k (0) \\
\vdots \\
V_n (0)
\end{bmatrix}
\]  

\[ (1.1) \]
Generally the short circuit currents are so large compared to the steady state loads, the loads may be neglected. However, the loads can be represented as impedances using the above voltages for example, the load \( S_i \) may be approximated by the impedance

\[
Z_i = \frac{|V_i(0)|^2}{S_i^*}
\]  

(1.2)

The changes in network voltage due to the fault are found from the system with all voltages shorted to ground and the prefault voltage \( V_k(0) \) applied as shown in the figure below:

Note the capacitors for the pi equivalent of line \( ik \). Also note the loads are replaced by the approximation of impedances. The voltage source at \( i \) is grounded and its impedance remains as shown above. The bus voltage changes caused by the fault in this circuit are represented by the column vector

\[
\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}
\]  

(1.3)

From Thevenin's theorem the voltages during the fault are found from

\[
V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}
\]  

(1.4)

We also know for the nodal equations that

\[
I_{bus} = Y_{bus}V_{bus}
\]  

(1.5)

where \( I_{bus} \) is the bus current entering the bus and \( Y_{bus} \) is the bus admittance matrix. The diagonal elements of each bus is the sum of the admittances connected to that bus, i.e.

\[
Y_{ii} = \sum_{j=0}^{m} y_{ij} \quad j \neq i
\]  

(1.6)
The off-diagonal element is equal to the negative of the admittance between the nodes, i.e.

\[ Y_{ij} = Y_{ji} = -y_{ij} \]  

(1.7)

where \( y_{ij} \) is the actual admittance between nodes \( i \) and \( j \). For the Thevenin circuit above, the nodal equations are

\[
\begin{bmatrix}
0 & \cdots & Y_{1k} & \cdots & Y_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
-Y_k(F) & \vdots & Y_{k1} & \cdots & Y_{kn} \\
0 & \cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\begin{bmatrix}
\Delta V_1 \\
\vdots \\
\Delta V_k \\
\vdots \\
\Delta V_n \\
\end{bmatrix}
= \begin{bmatrix}
\Delta V_k \\
\vdots \\
\Delta V_k \\
\vdots \\
\Delta V_k \\
\end{bmatrix}
\]  

(1.8)

Note that the minus sign is due to the fact the fault current is shown leaving node \( k \). The above matrix equation can be written as

\[ I_{bus}(F) = Y_{bus}\Delta V_{bus} \]  

(1.9)

which can be solved for the voltage change thus

\[ \Delta V_{bus} = Z_{bus}I_{bus}(F) \]  

(1.10)

where \( Z_{bus} = Y_{bus}^{-1} \) is known as the bus impedance matrix (not to be mixed-up with the impedance matrix in mesh equations of circuit theory!)

Using (1.10) in (1.4) we have:

\[ V_{bus}(F) = V_{bus}(0) + Z_{bus}I_{bus}(F) \]  

(1.11)

Which can be expanded into matrix form:

\[
\begin{bmatrix}
V_1(F) \\
\vdots \\
V_k(F) \\
\vdots \\
V_n(F) \\
\end{bmatrix}
= \begin{bmatrix}
V_1(0) \\
\vdots \\
V_k(0) \\
\vdots \\
V_n(0) \\
\end{bmatrix}
+ \begin{bmatrix}
Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1n} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kn} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{n1} & \cdots & Z_{nk} & \cdots & Z_{nn} \\
\end{bmatrix}
\begin{bmatrix}
0 \\
\vdots \\
I_k(F) \\
\vdots \\
0 \\
\end{bmatrix}
\]  

(1.12)

Using the \( k \)th equation we have:

\[ V_k(F) = V_k(0) - Z_{kk}I_k(F) \]  

(1.13)

Also it is clear that

\[ V_k(F) = Z_f I_k(F) \]  

(1.14)

From (1.13) and (1.14) we have:

\[ I_k(F) = \frac{V_k(0)}{Z_{kk} + Z_f} \]  

(1.15)

Thus we can solve for any bus voltage using (1.12) for row \( i \):

\[ V_i(F) = V_i(0) - Z_{ik}I_k(F) \]  

(1.16)

And using (1.15) in (1.16) we have:

\[ V_i(F) = V_i(0) - Z_{ik} \frac{V_k(0)}{Z_{kk} + Z_f} \]  

(1.17)
Knowing all the bus voltages during a fault we can solve for all the fault currents, in particular, for line current from bus $i$ to bus $j$ we have:

$$I_{ij}(F) = \frac{V_i(F) - V_j(F)}{z_{ij}}$$ (1.18)

It is noted that this method works for any bus under fault (i.e. the faulted bus $k$ could be any one bus of the buses in the system). [Note that $z_{ij}$ is the impedance between bus $i$ and bus $j$. It is not the $i$-$j$-th element of $Z_{bus}$.]

The method suggested above to find the bus impedance matrix by inversion of the bus admittance matrix is not feasible for very large power systems. An alternate method for the direct formation (or building) of the matrix $Z_{bus}$ will be discussed in the next section.

Example 9.2

A three-phase fault with a fault impedance of $Z_f = j0.16\text{pu}$ occurs at bus 3 in the network for Example 9.1. Using the bus impedance matrix method, compute the fault current, the bus voltages, and the line currents during the fault.

First the network in Example 9.1 is redrawn to the right using impedances. By inspection we can find the $Y_{bus}$ thus:

$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5 \end{bmatrix}$$

Note that this is the $Y_{bus}$ before the fault occurs. By inversion using Matlab we have:

```matlab
Ybus = j*[-8.75  1.25  2.5  
        1.25 -6.25  2.5  
        2.5  2.5  -5.0];
Zbus = inv(Ybus);
```

From equation (1.15) we have:

```matlab
Zf = j*.16;
V0=[1; 1; 1];
I3F = V0(1)/(Zbus(3,3)+Zf)
```

$$I_{3F} =$$
and from (1.16) we have:

\[ \mathbf{V}_F = \mathbf{V}_0 - \mathbf{I}_3 \mathbf{Z}_{bus}(::,3) \]

\[ \mathbf{V}_F = \begin{bmatrix} 0.7600 \\ 0.6800 \\ 0.3200 \end{bmatrix} \]

Finally from (1.17) we have:

\[
\begin{align*}
\mathbf{z}_{12} &= j \cdot 0.8; \\
\mathbf{z}_{13} &= j \cdot 0.4; \\
\mathbf{z}_{23} &= j \cdot 0.4;
\end{align*}
\]

\[
\begin{align*}
\mathbf{I}_{12} &= (\mathbf{V}_F(1) - \mathbf{V}_F(2))/\mathbf{z}_{12} \\
\mathbf{I}_{13} &= (\mathbf{V}_F(1) - \mathbf{V}_F(3))/\mathbf{z}_{13} \\
\mathbf{I}_{23} &= (\mathbf{V}_F(2) - \mathbf{V}_F(3))/\mathbf{z}_{23}
\end{align*}
\]

\[\mathbf{I}_{12} = \begin{bmatrix} 0 - 0.1000i \\ \end{bmatrix} \]

\[\mathbf{I}_{13} = \begin{bmatrix} 0 - 1.1000i \\ \end{bmatrix} \]

\[\mathbf{I}_{23} = \begin{bmatrix} 0 - 0.9000i \\ \end{bmatrix} \]

These results are identical to those found earlier in Example 1. Note that the need for the repeated simplification to find the Thevenin impedance is removed, but a matrix inversion is introduced. This inversion will also be eliminated by the direct building method of the bus impedance matrix to be discussed in the next section.